



GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY

SPECIAL FUNCTIONS AND COMPLEX VARIABLES

Course Code: GR15A2058
II Year I Semester

L:2 T:1 P:0 C:3

Prerequisites: Co-ordinate Geometry, Calculus, Linear Differential Equations

Course Objectives

1. To learn Special functions like Legendre, Bessel, Chebyshev functions.
2. To introduce Complex functions and their qualitative behaviour in the Complex plane.
3. To understand Singularity and Residue theory.

Course Outcomes: At the end of the course, the student will be able to

1. Solve Linear Differential Equations using Power-Series Methods.
2. Approximate polynomials in terms of Legendre, Bessel, Chebyshev.
3. Solve Real Definite Integrals using Cauchy's Residue Theory.

Unit-I

Special Functions I: Solution to Cauchy-Euler Problem. Introduction to series solution of differential equations. Legendre polynomials (as solution of second order differential equation) – properties – Rodrigue's formula – recurrence relations – orthogonality.

Unit-II

Special Functions-II: Bessel Functions – properties – recurrence relations – orthogonality. Chebyshev polynomials (as solution of second order differential equation) – properties – recurrence relations – orthogonality.

Unit-III

Functions of a Complex variable: Continuity – differentiability – Analyticity – Cauchy-Riemann equations – Maxima-minima principle – Harmonic and conjugate harmonic functions – Milne-Thompson method.

Elementary functions: General power Principal value. Logarithmic function.

Conformal mapping: Transformations e^z , $\text{Im } z$, Z^2 , Z^n (n is a positive integer), $\sin z$, $\cos z$, $z + (a/z)$. Translation, rotation, inversion and bilinear transformation – fixed point – cross ratio – invariance of circles and cross ratio – determination of bilinear transformation mapping of 3 given points



Unit-IV

Complex integration: Line integral – evaluation along a path – Cauchy's integral theorem – Cauchy's integral formula – Generalized integral formula.

Complex power series

Radius of convergence : Expansion in Taylor series, Maclaurin's series, Laurent series.

Unit-V

Singular points, Residues and Applications of Complex Integration:

Singular points – isolated singular point – pole of order m – essential singularity. (Distinction between real analyticity and complex analyticity). Residue – Evaluation of residue by formula and by Laurent series – Residue theorem.

Evaluation of real integrals of the types

(a) Improper integrals $\int_{-\infty}^{\infty} f(x)dx.$

(b) $\int_c^{c+2\pi} f(\cos \theta, \sin \theta) d\theta.$

(c) $\int_{-\infty}^{\infty} f(x) * \cos(mx) dx.$

(d) Integrals by indentation.

Teaching methodologies

1. Tutorial sheets uploaded in website
2. NPTEL video lectures
3. MATLAB exercises for visualization

Text Books

1. Advanced Engineering Mathematics: R.K.Jain and S.R.K.Iyengar - Narosa Publishing House
2. Advanced Engineering Mathematics: Erwin Kreyszig-Wiley Publications

Reference Books

1. Schaum's Outline series on complex variables.
2. Higher Engineering Mathematics: B.S. Grewal, Khanna Publications